Die dichtesten Gitterpunkte im Raum

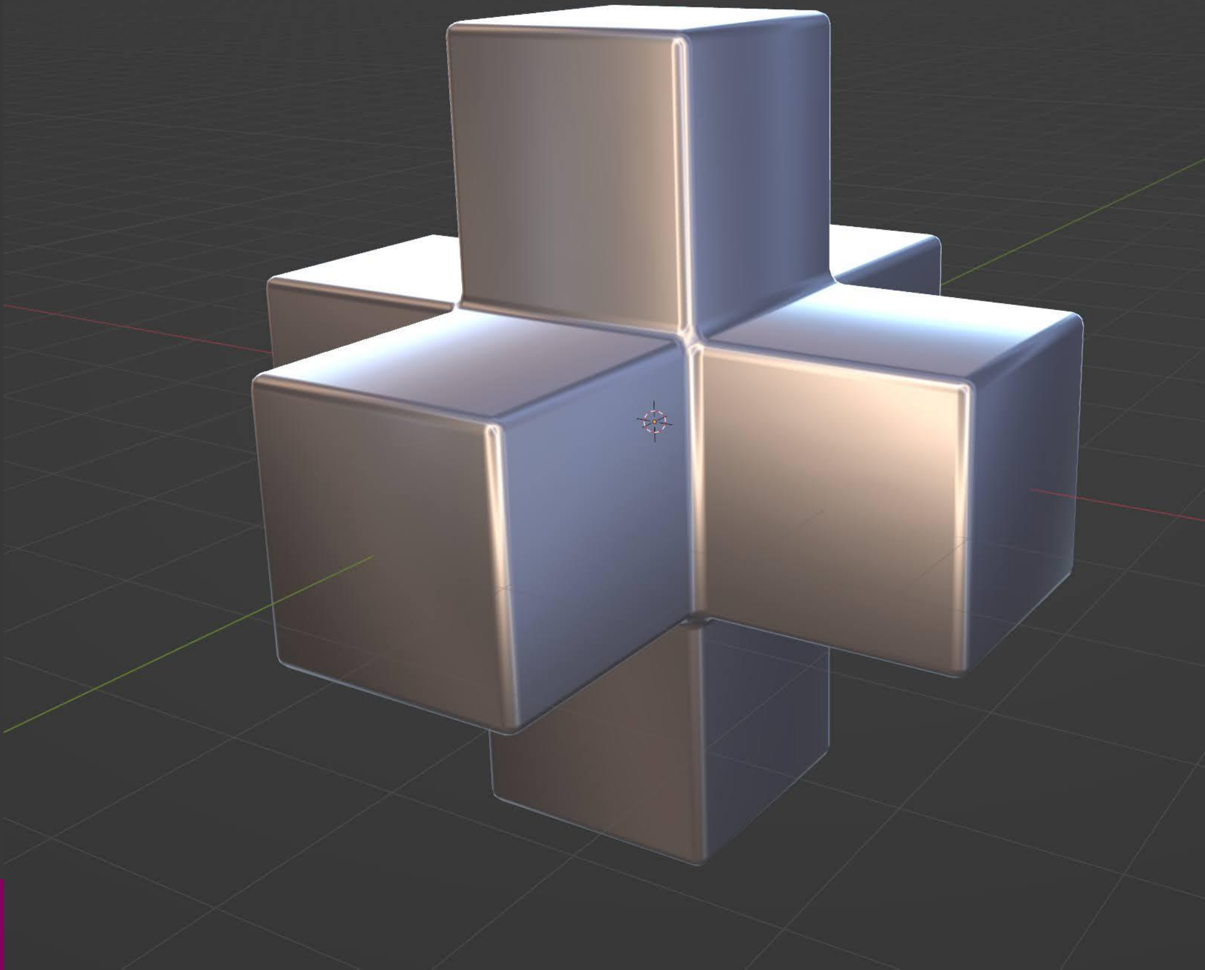
Nook 2025, Fiona Runge

Kurz was zum Hintergrund

Was ist hier los, und wie kam es dazu?

3D Plusse

- 2023:
  Raum füllend?



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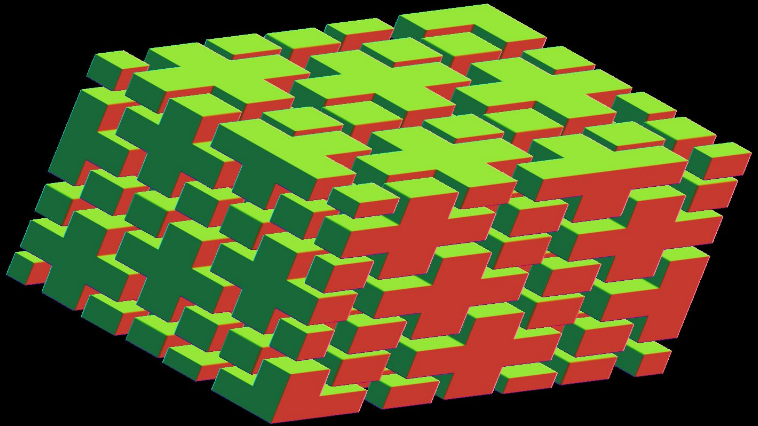
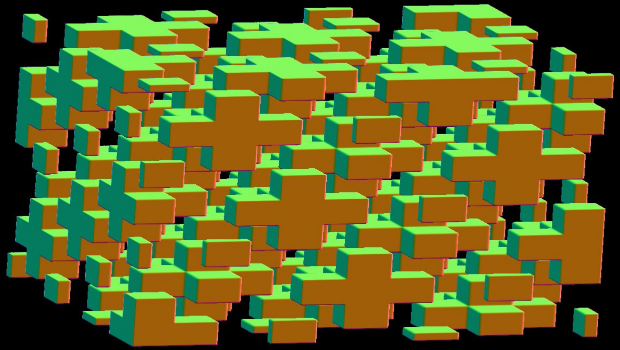
3D Plusse

- 2024:
  Malen nach Zahlen





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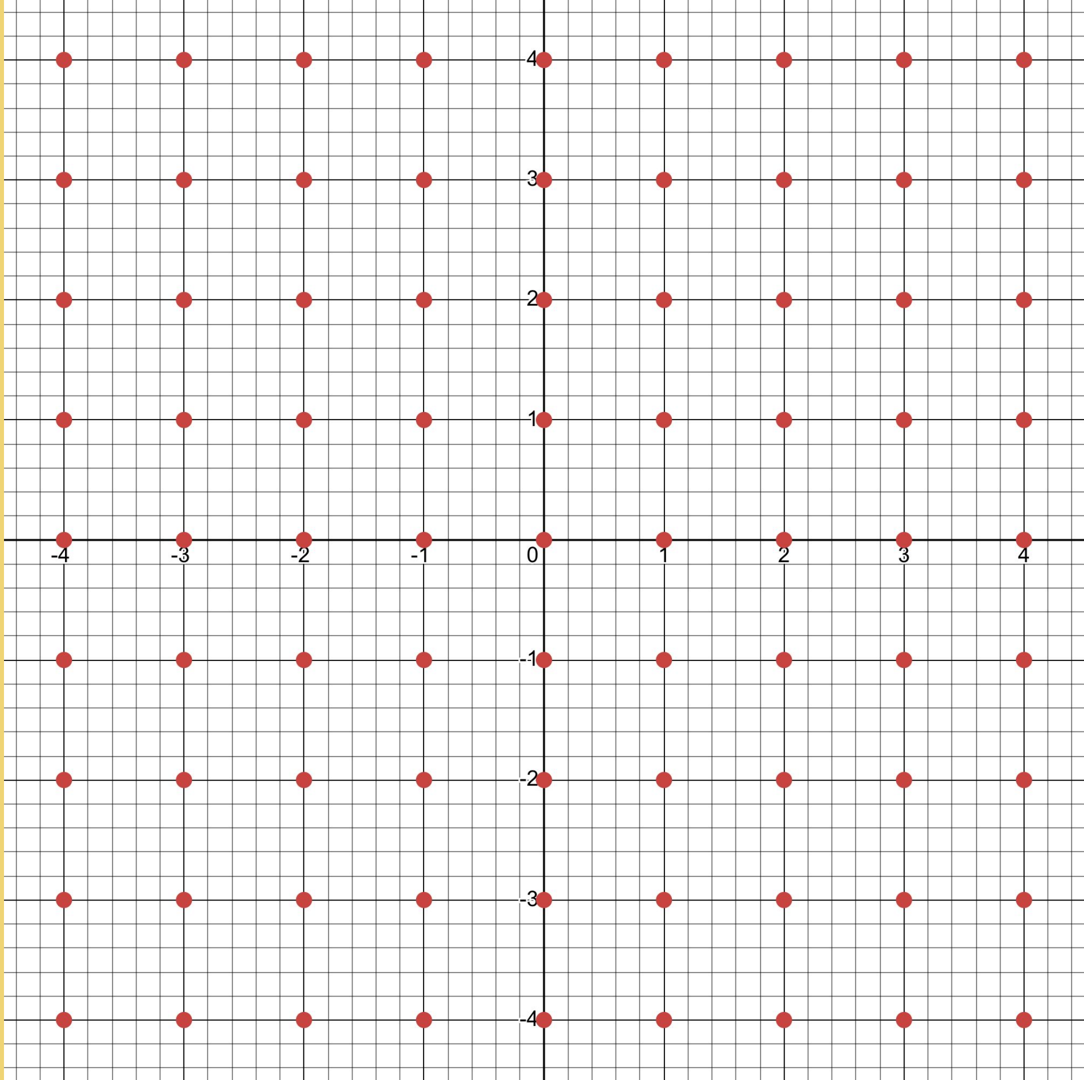
Sind diese Gitterpunkte hier im Raum mit uns?

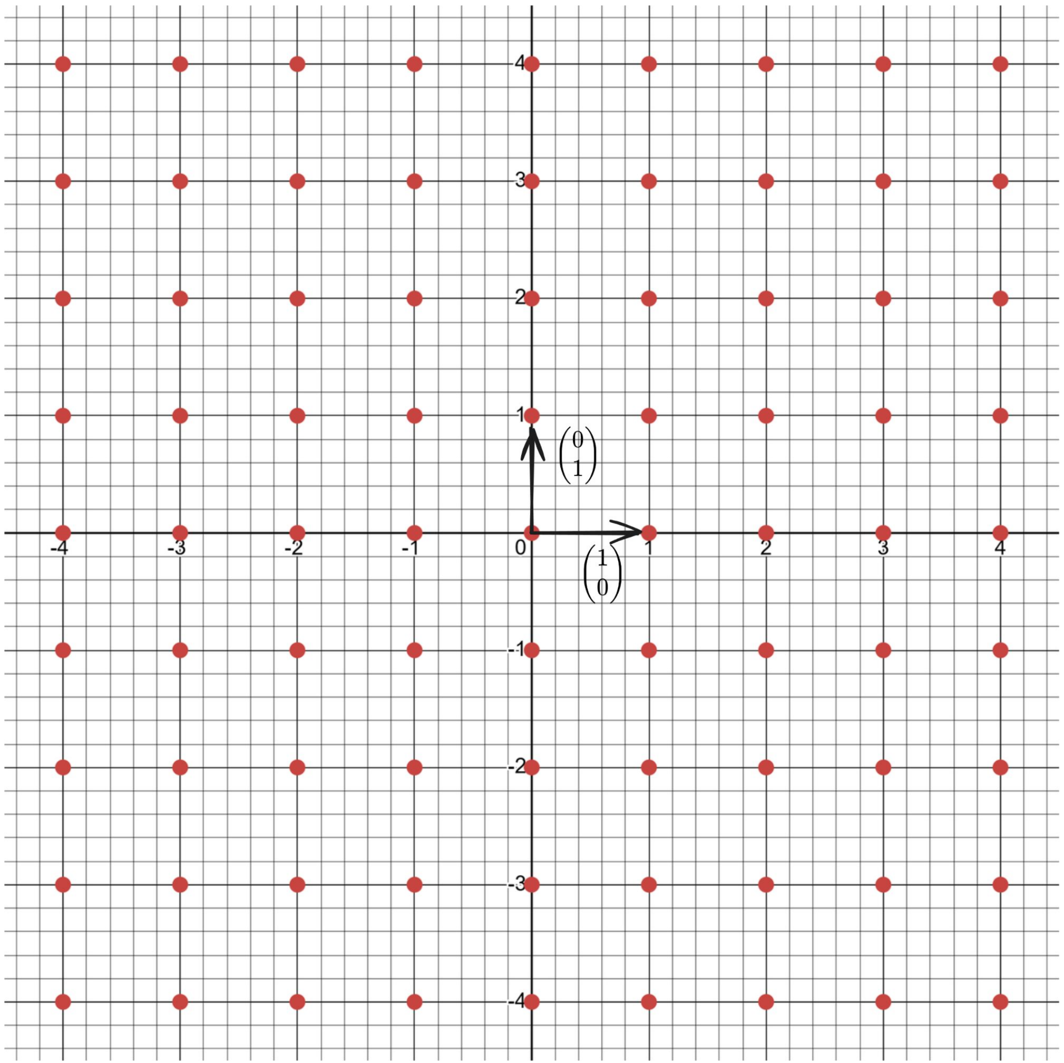
Beispiel 1

- 2D



Beispiel 1

- 2D

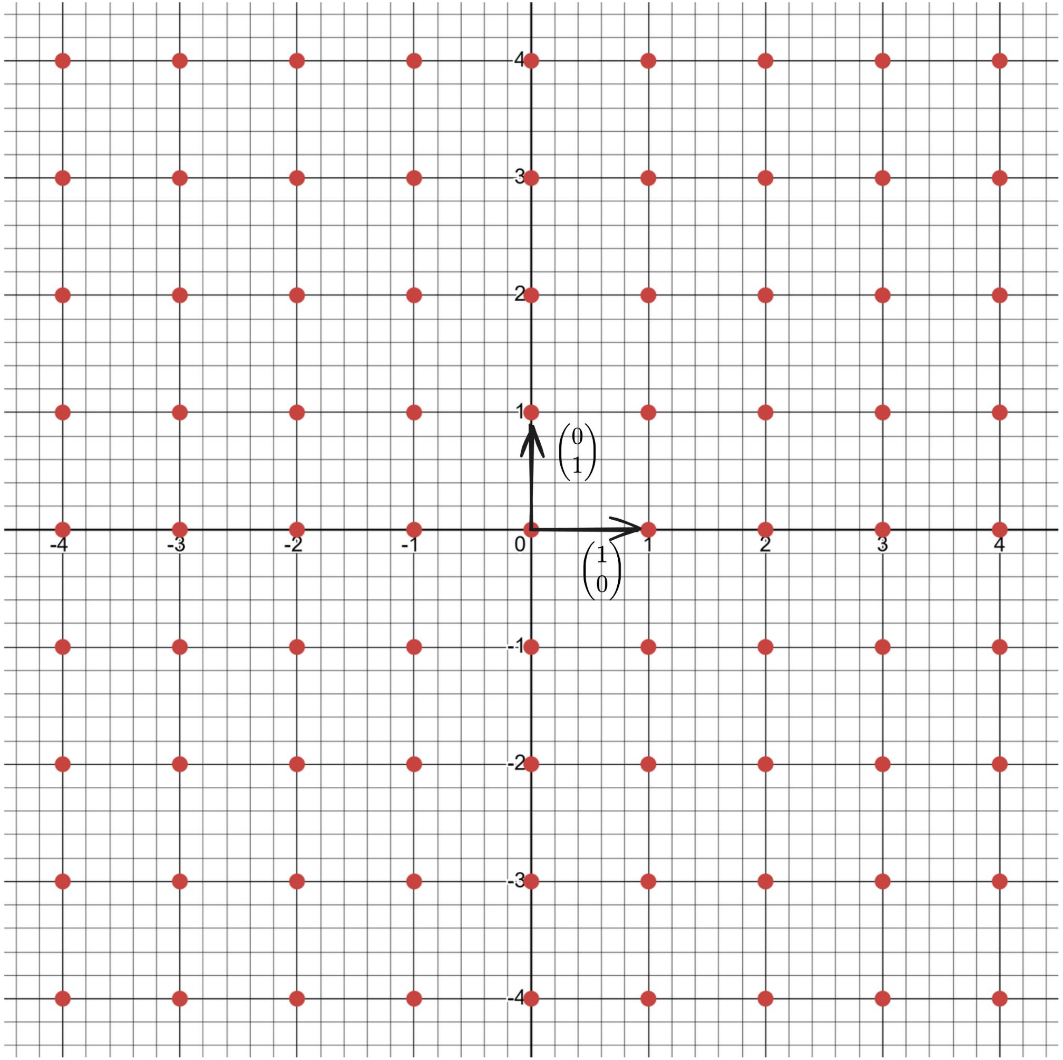


Beispiel 1

- 2D
- Linearkombination:

$$p = x \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

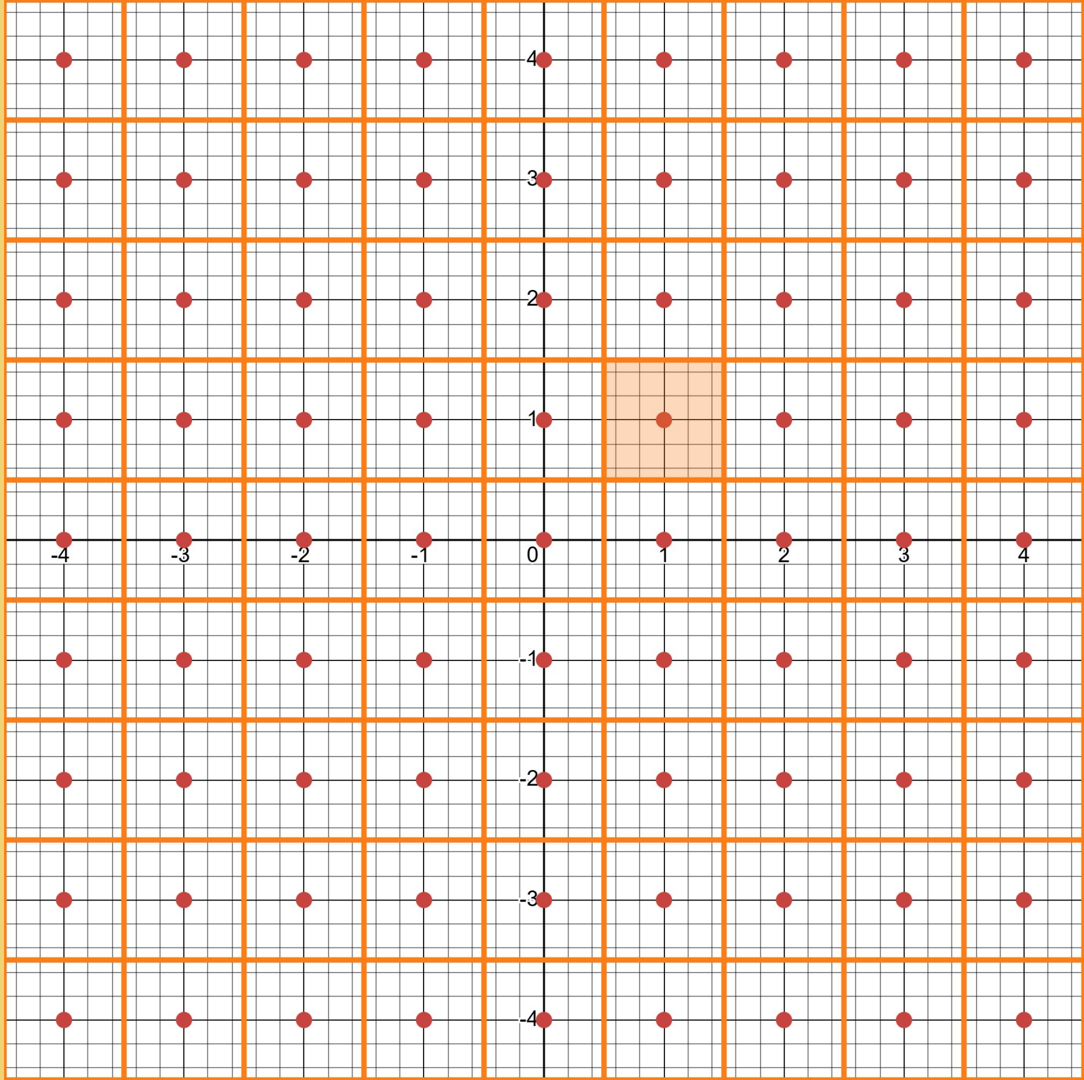
$$x, y \in \mathbb{Z}$$



Was soll das heißen, die ‘dichtesten’ Punkte?

Beispiel 1

- Es gibt so ‘Zellen’
- $\begin{pmatrix} \lfloor x \rceil \\ \lfloor y \rceil \end{pmatrix}$

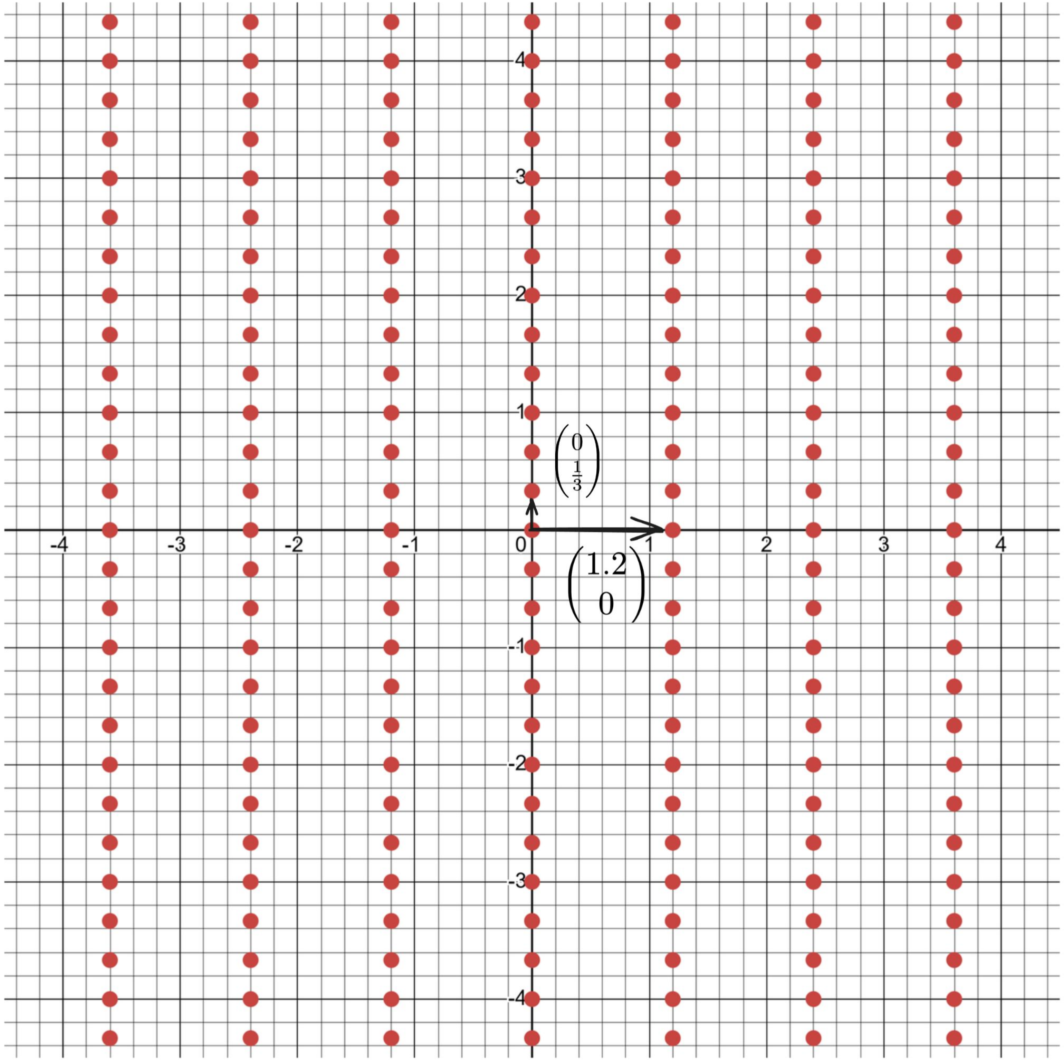


Beispiel 2

- Schrittgrößen
-

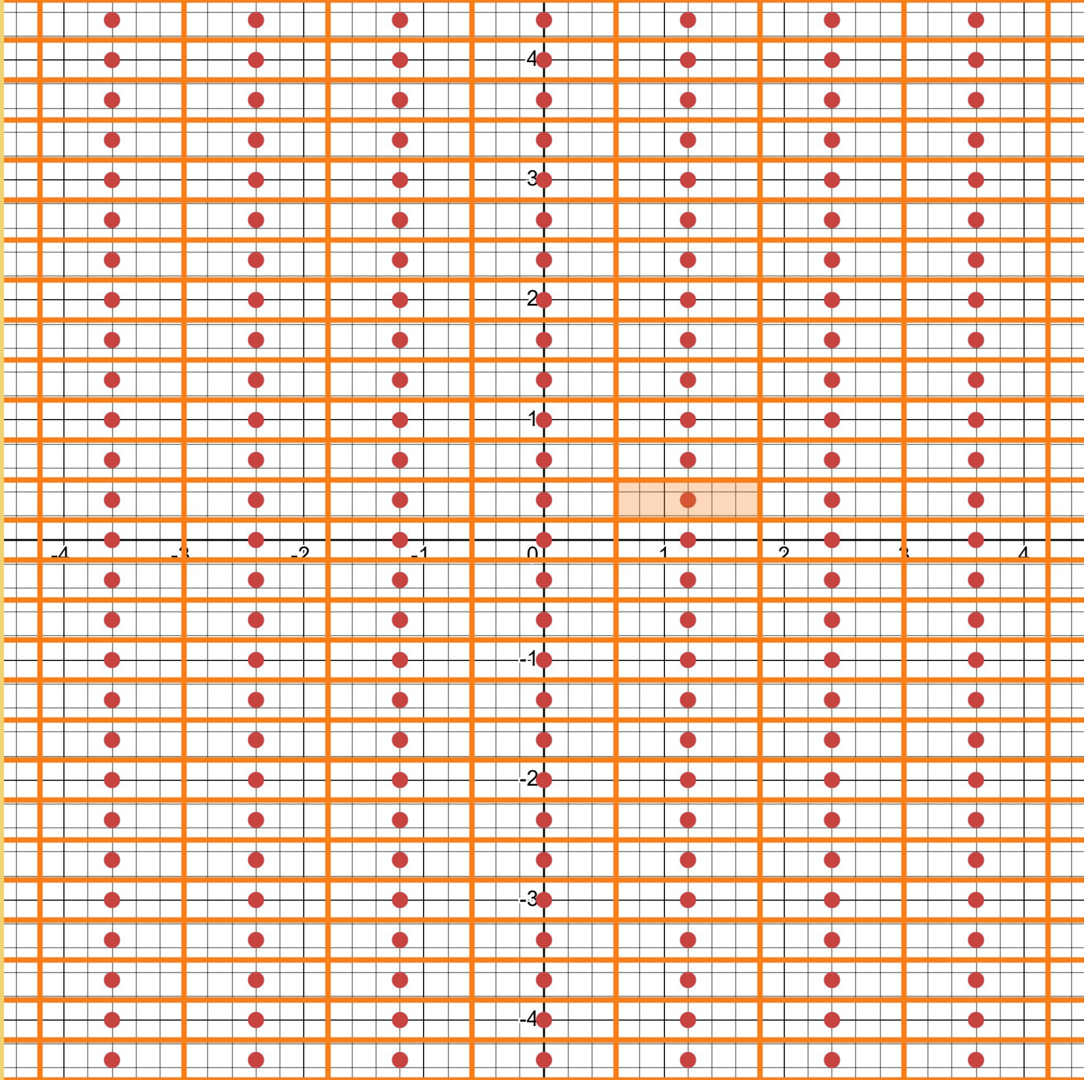
$$p = \begin{pmatrix} \frac{6}{5} & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x, y \in \mathbb{Z}$$



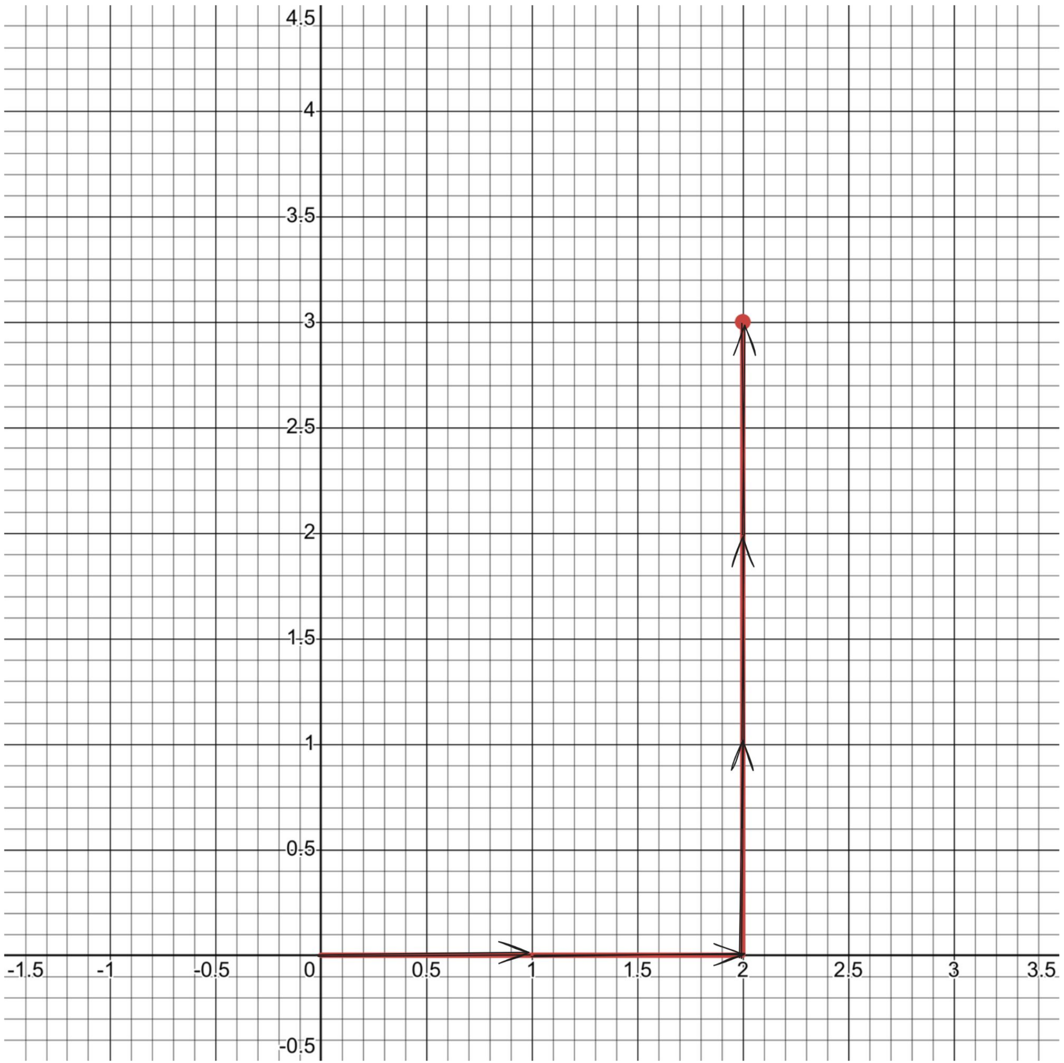
Beispiel 2

- 🤔
- Mehr als Runden?
- Wir brauchen nen Trick.



Basis

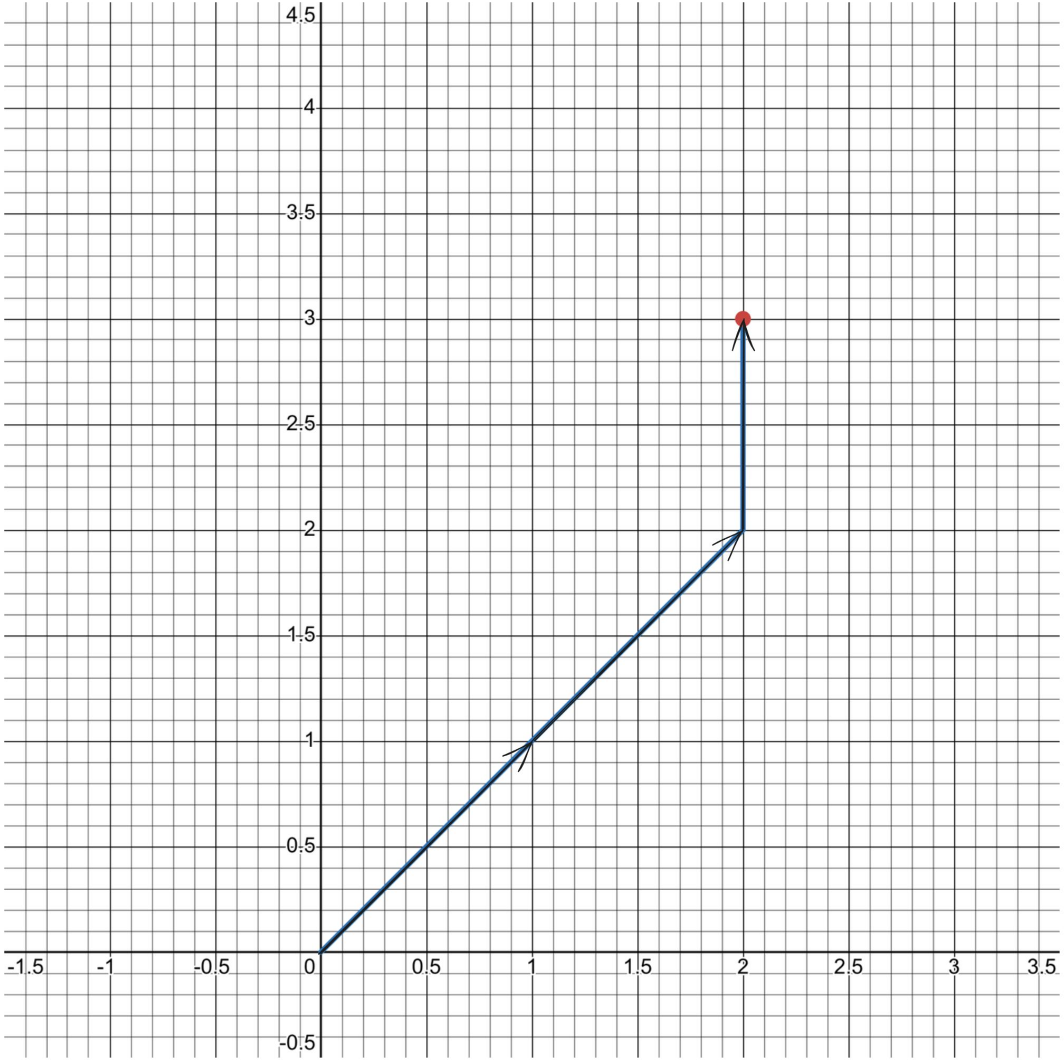
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$



Basis

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

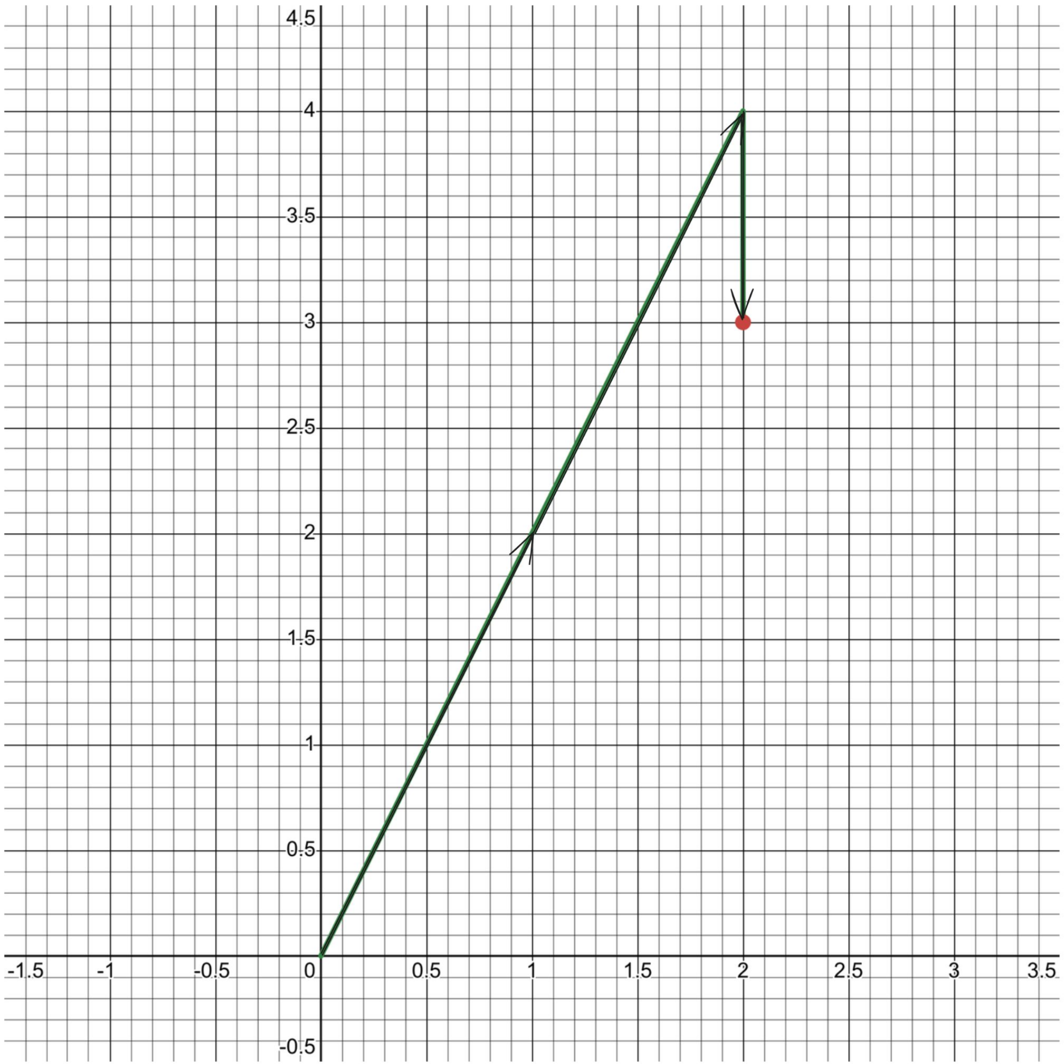


Basis

$$\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \cdot \begin{pmatrix}2\\3\end{pmatrix}$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}1 & 0\\1 & 1\end{pmatrix} \cdot \begin{pmatrix}2\\1\end{pmatrix}$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}1 & 0\\2 & 1\end{pmatrix} \cdot \begin{pmatrix}2\\-1\end{pmatrix}$$



Basiswechsel

$$m = \begin{pmatrix} \frac{6}{5} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}, m^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$m \times m^{-1} = I_2$$

Basiswechsel

$$m = \begin{pmatrix} \frac{6}{5} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}, m^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

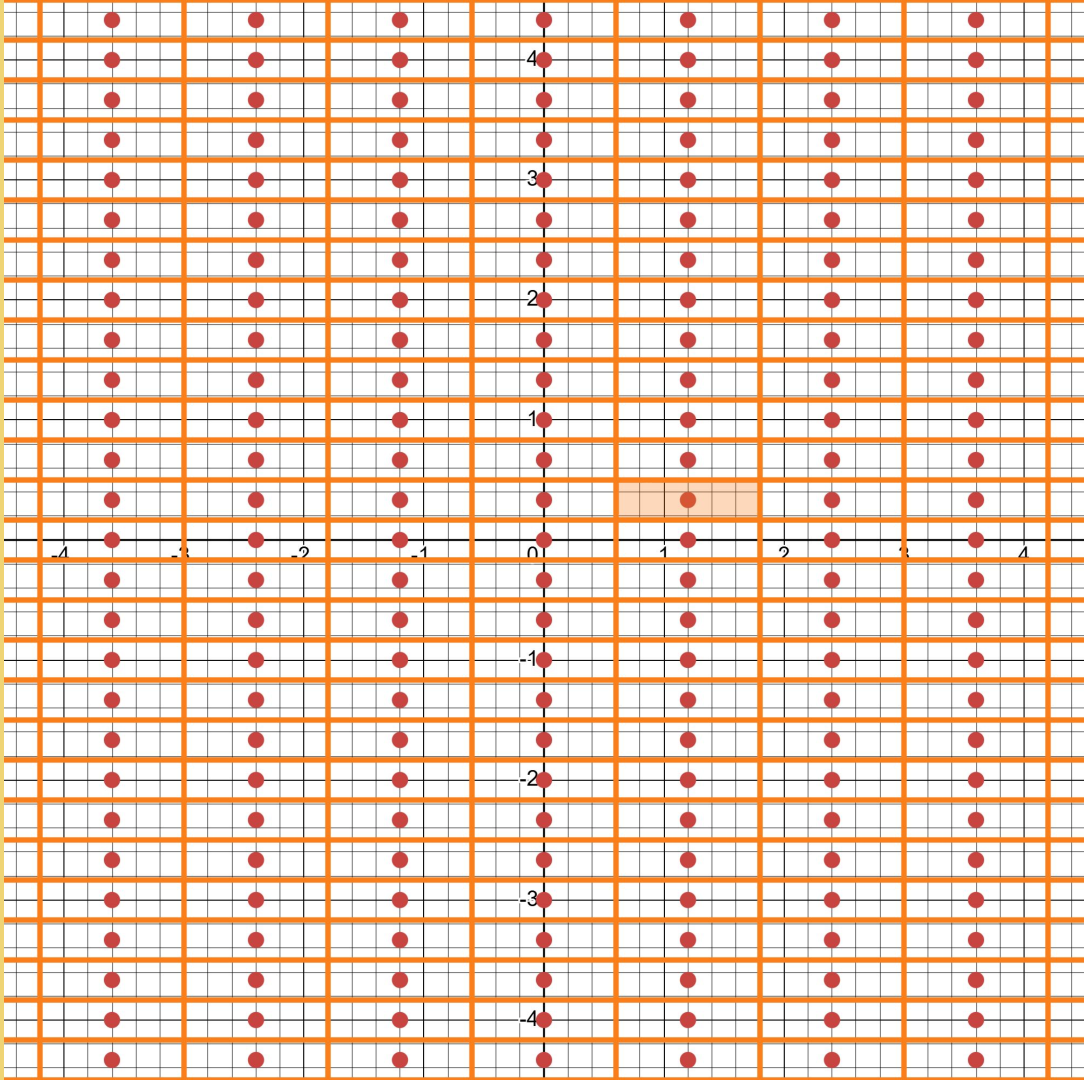
$$m \times m^{-1} = I_2$$

$$\begin{pmatrix} \frac{6}{5}a + 0c & \frac{6}{5}b + 0d \\ 0a + \frac{1}{3}c & 0b + \frac{1}{3}d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$m^{-1} = \begin{pmatrix} \frac{5}{6} & 0 \\ 0 & 3 \end{pmatrix}$$

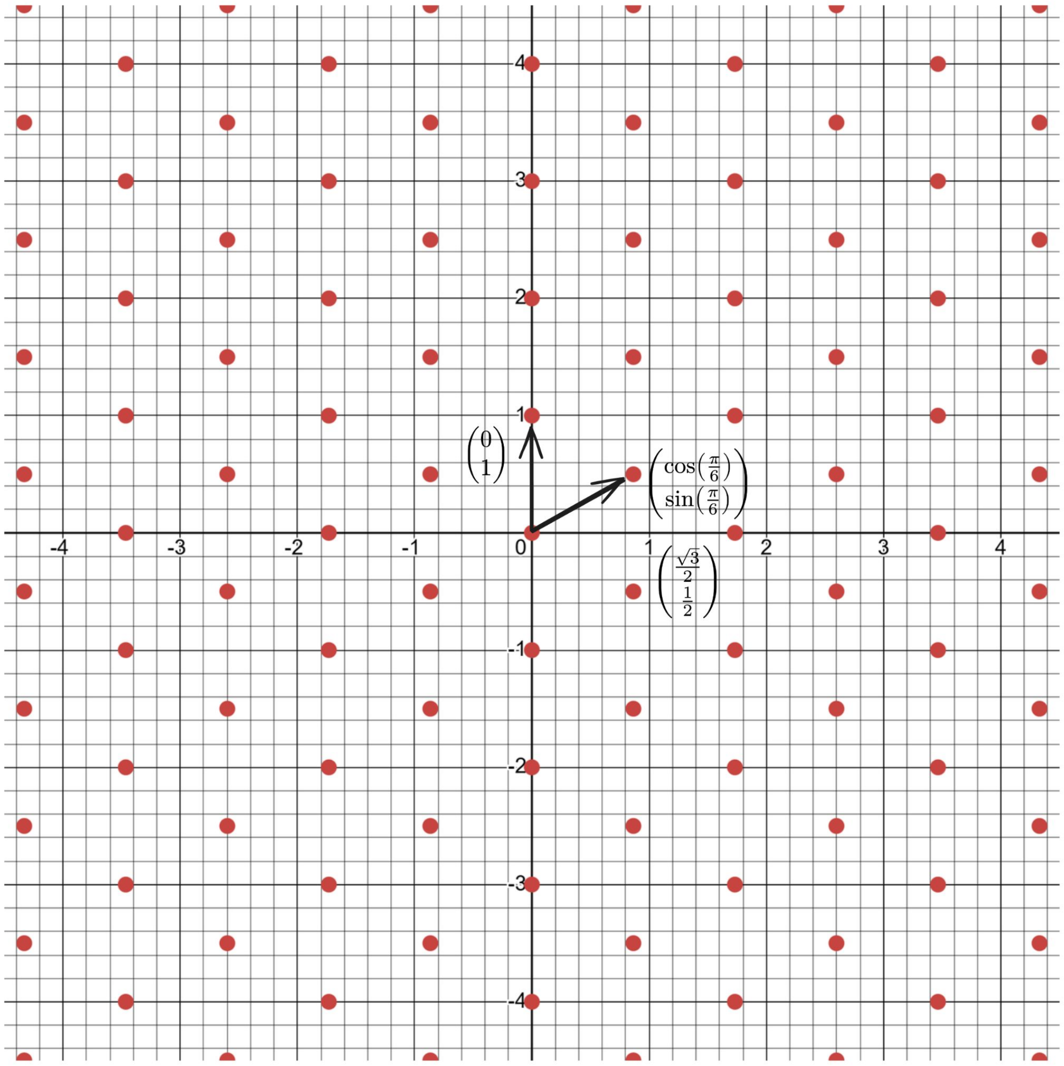
Beispiel 2

$$m \cdot \lfloor m^{-1} \cdot p \rceil$$



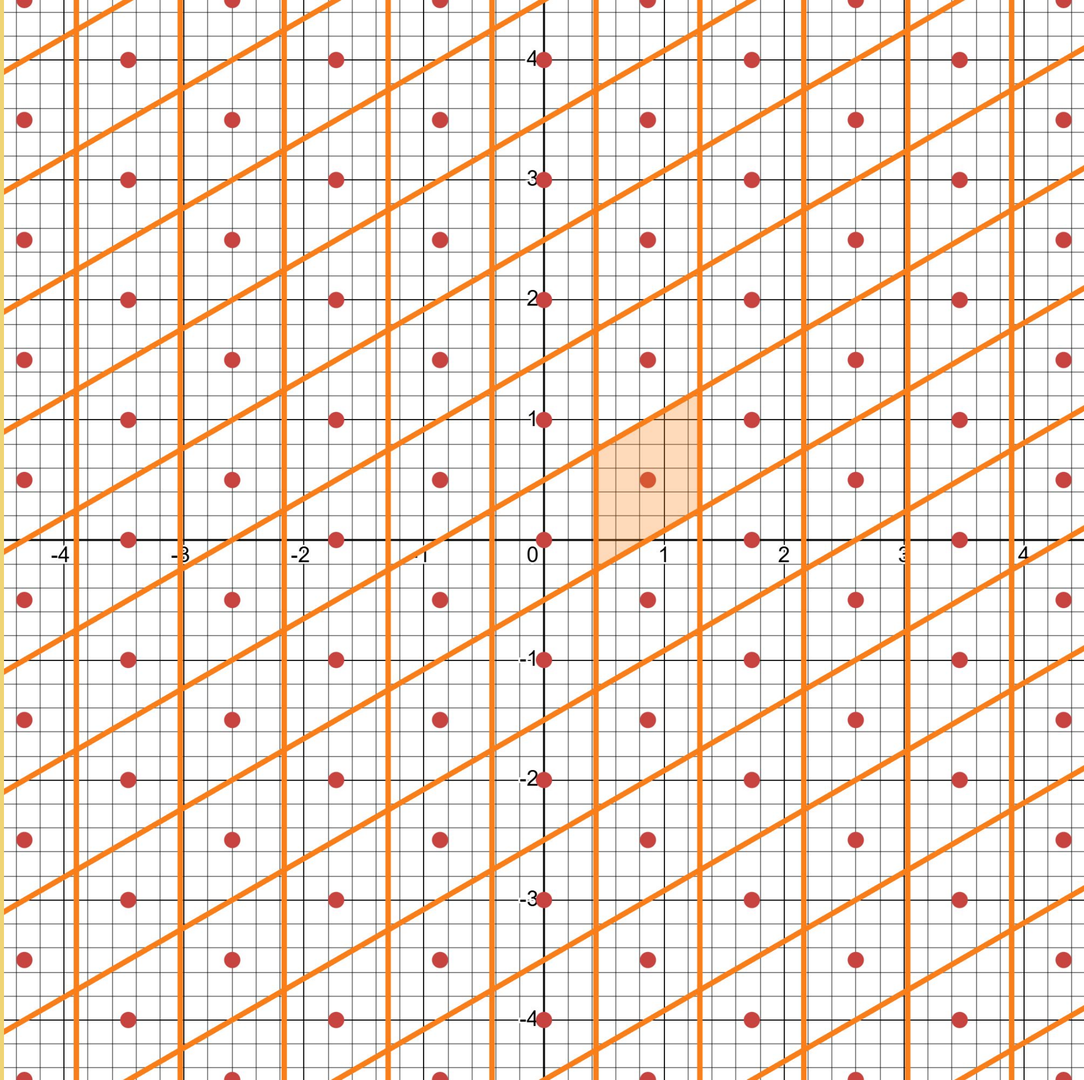
Beispiel 3

- Schiefes Gitter
- $$p = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



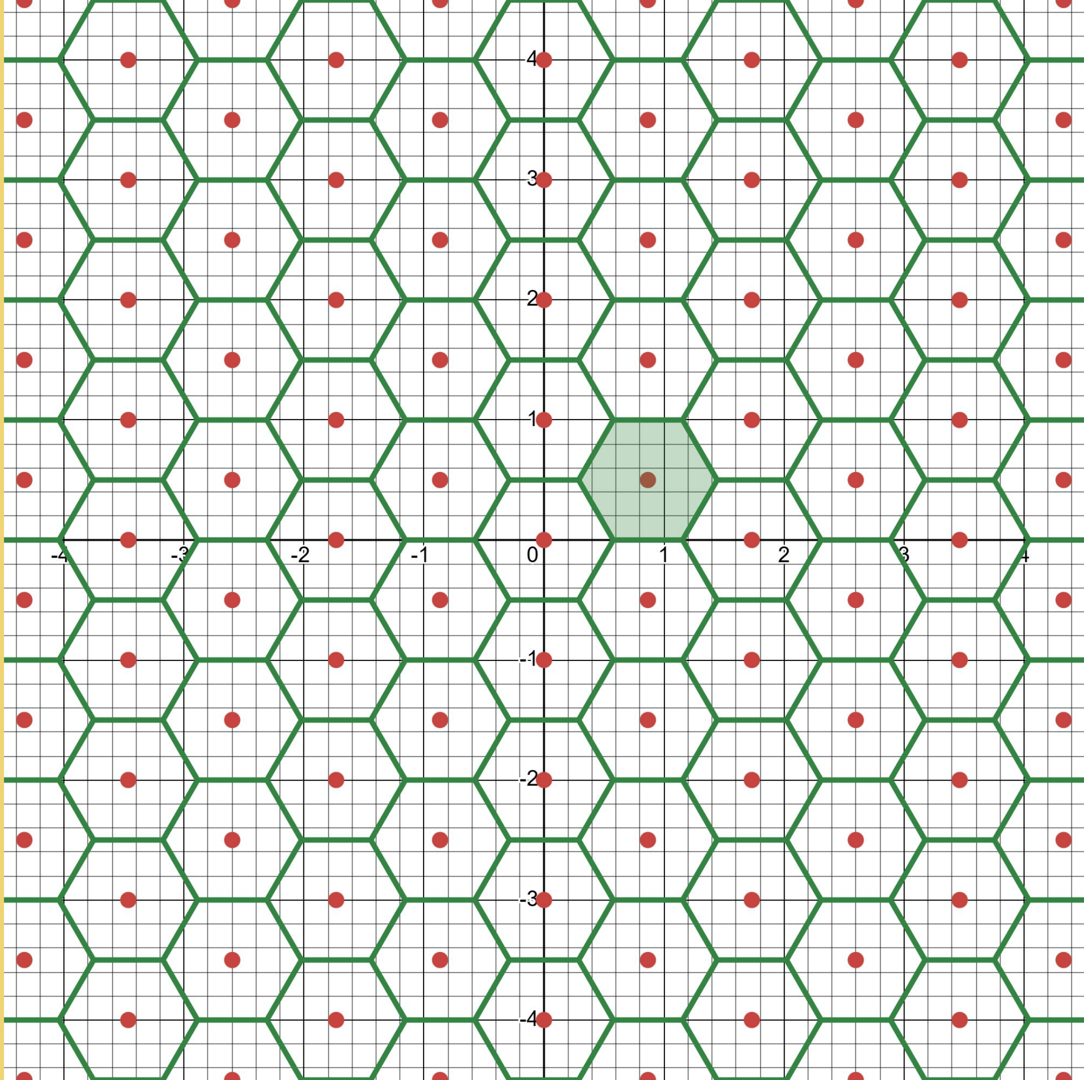
Beispiel 3

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \cdot \left\lfloor \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & 1 \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix} \right\rceil$$



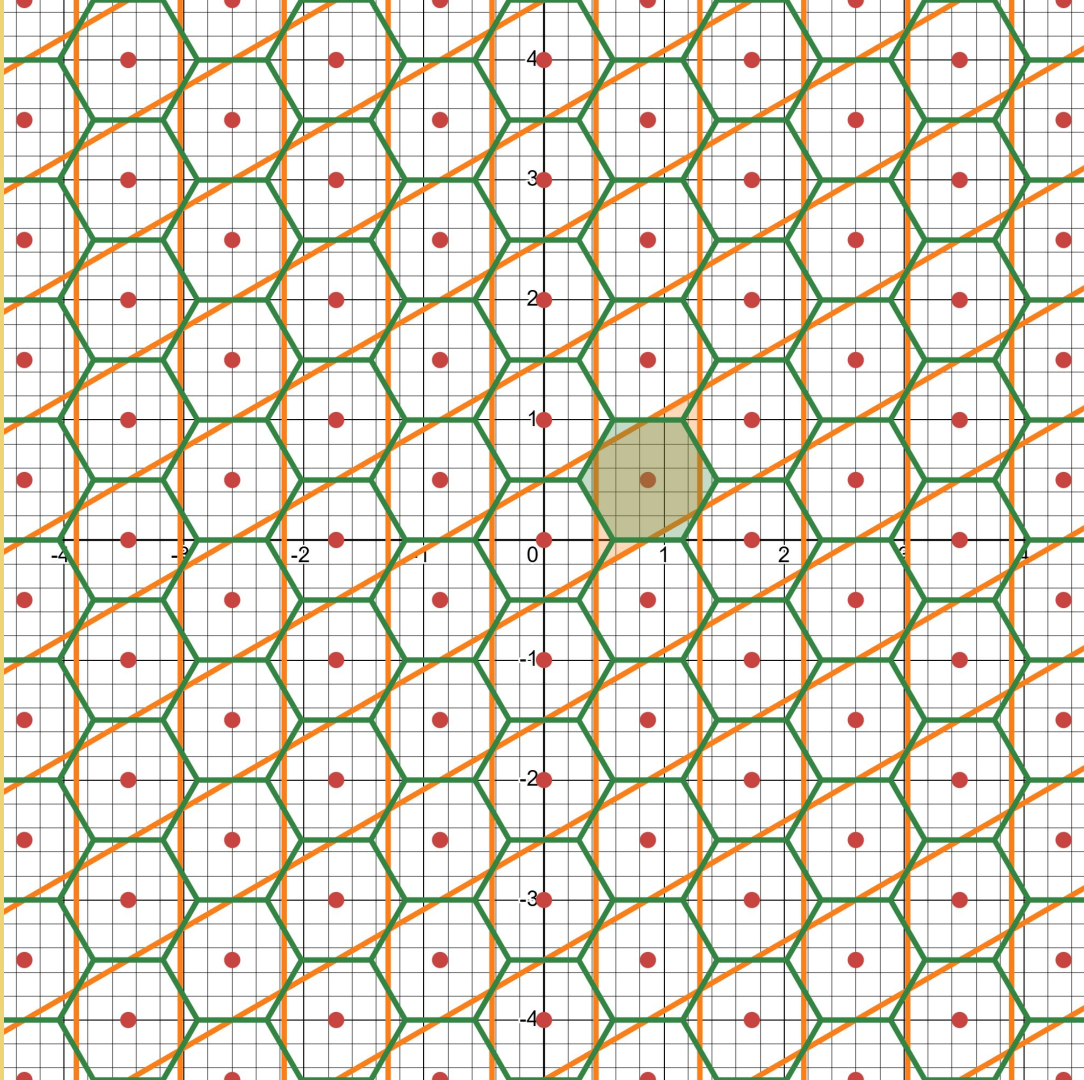
Beispiel 3

- 🤔muss das nicht eher so?



Beispiel 3

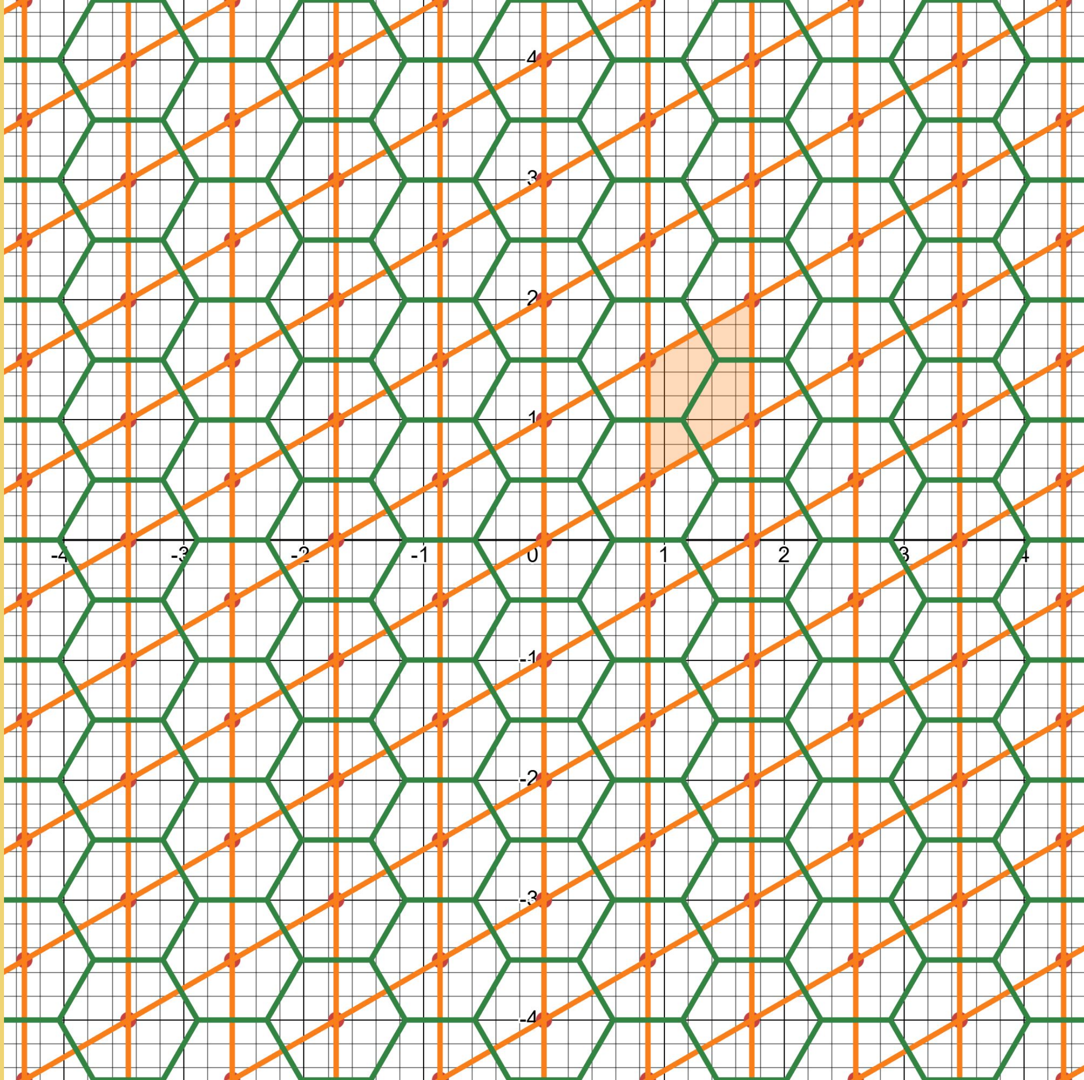
- 🤔muss das nicht eher so?
- Können wir das reparieren?



Beispiel 3

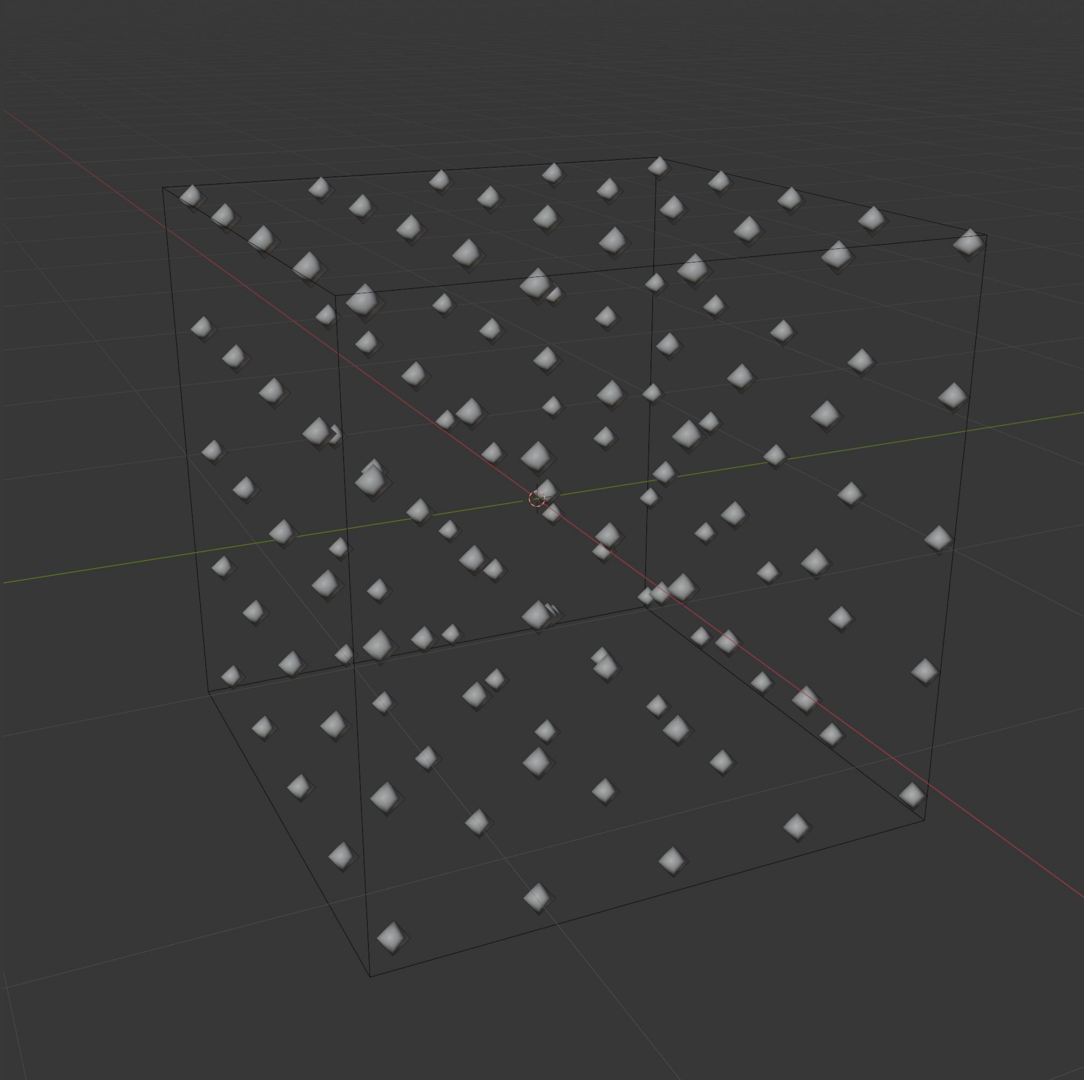
$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \left\{ \begin{pmatrix} \lfloor x \rfloor \\ \lfloor y \rfloor \end{pmatrix}, \begin{pmatrix} \lceil x \rceil \\ \lfloor y \rfloor \end{pmatrix}, \begin{pmatrix} \lfloor x \rfloor \\ \lceil y \rceil \end{pmatrix}, \begin{pmatrix} \lceil x \rceil \\ \lceil y \rceil \end{pmatrix} \right\}$$

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \cdot f\left(\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & 1 \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix} \right)$$



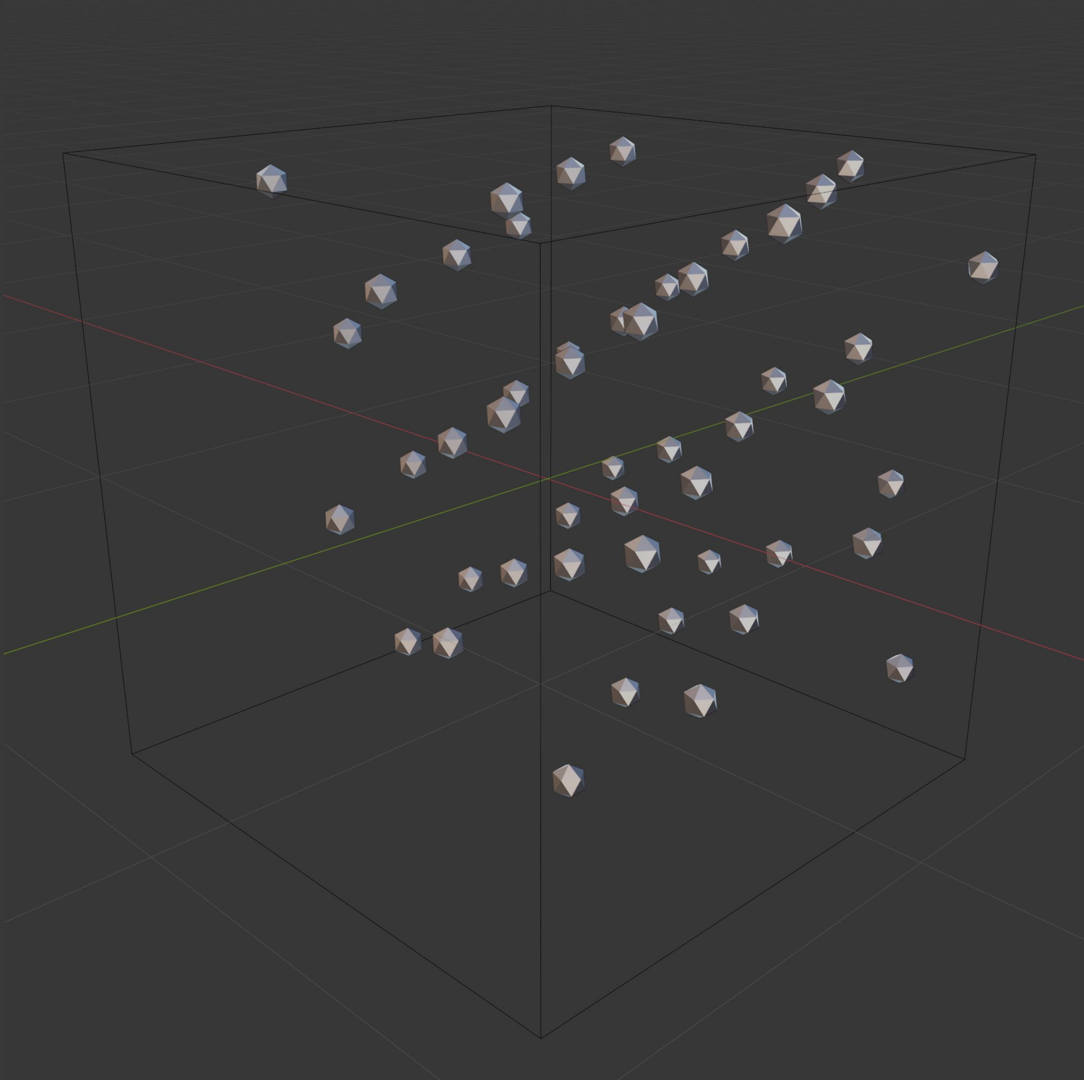
Beispiel 4

- 3D
- $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} y + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} z$



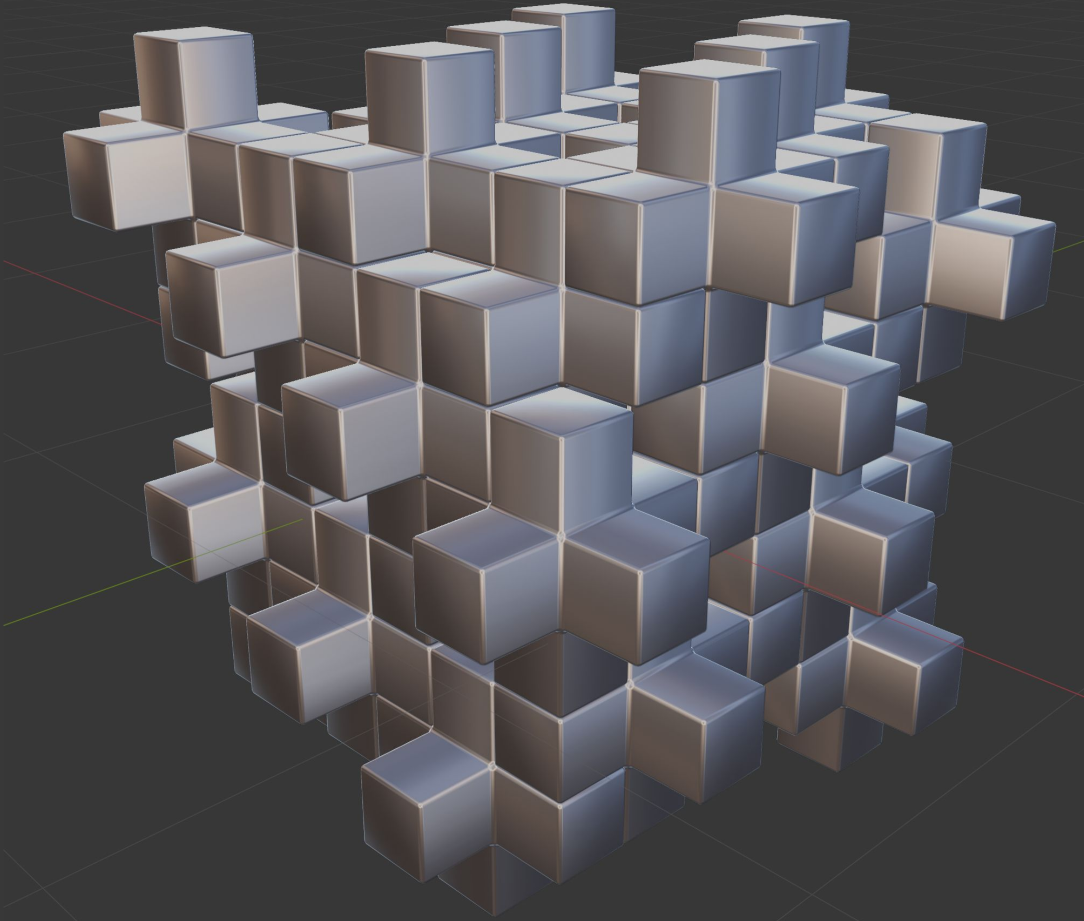
Beispiel 5

- 3D
- $$\begin{pmatrix} 1 & 0 & 1 \\ 5 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



Beispiel 5

- 3D
- $$\begin{pmatrix} 1 & 0 & 1 \\ 5 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$




Hallo
Nook
2025 ;)